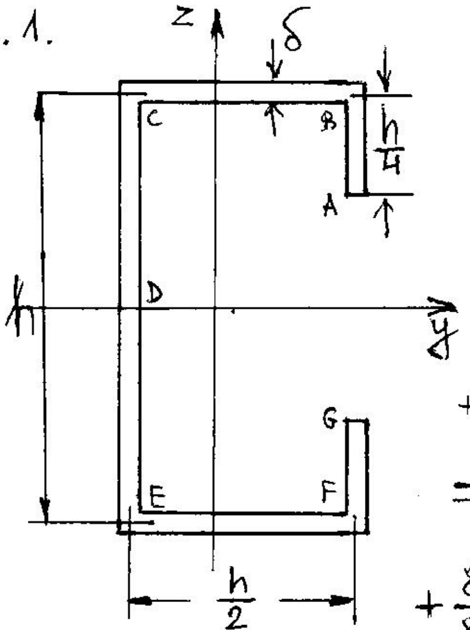
s. IV

Z. 1. Wyznaczyć położenie S.P. $\delta \ll h$

$\delta = \text{const}$.



$$J_y \approx \frac{h^3\delta}{12} + 2\cdot\left(\frac{h}{2}\delta\cdot\left(\frac{h}{2}\right)^2 + \frac{\left(\frac{h}{4}\right)^3\cdot\delta}{12} + \frac{h}{4}\delta\cdot\left(\frac{3h}{8}\right)^2\right) =$$

$$= \frac{h^3\delta}{12} + 2h^3\delta\left(\frac{1}{8} + \frac{1}{4\cdot16\cdot12} + \frac{9}{4\cdot8\cdot8}\right) = \frac{h^3\delta}{12} +$$

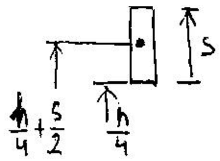
$$+ h^3\delta\left(\frac{1}{4} + \frac{1}{2\cdot16\cdot12} + \frac{9}{2\cdot8\cdot8}\right) = \frac{h^3\delta}{4}\left(1 + \frac{1}{3} + \frac{1}{8\cdot12} + \frac{9}{2\cdot8\cdot8}\right)$$

$$= \frac{h^3\delta}{4}\left(\frac{4}{3} + \frac{8}{8\cdot3\cdot2\cdot2\cdot8} + \frac{9\cdot2\cdot3}{2\cdot8\cdot8\cdot2\cdot3}\right) = \frac{h^3\delta}{4}\left(\frac{4\cdot8\cdot8\cdot2\cdot2}{3\cdot8\cdot8\cdot2\cdot2} + \right.$$

$$\left. + \frac{8+54}{8\cdot3\cdot2\cdot2\cdot8}\right) = \frac{h^3\delta}{4}\cdot\frac{\cancel{1086}\ \cancel{362}\ 181}{\cancel{12}\cdot2\cdot\cancel{3}\cdot8\cdot8} = 0.3535\, h^3\delta$$

$S_y(s)$:

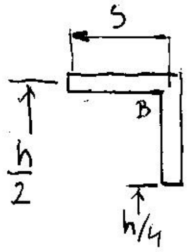
AB:



$$S_y(s) = s\cdot\delta\cdot\left(\frac{h}{4} + \frac{s}{2}\right) = \frac{s\delta}{4}(h + 2s),\quad S_y(A) = 0$$

$$S_y(B) = S_y\left(\frac{h}{4}\right) = \frac{h\cdot\delta}{4\cdot4}\left(h + 2\cdot\frac{h}{4}\right) = \frac{3}{32}h^2\delta$$

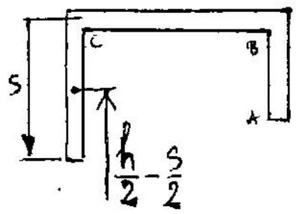
BC:



$$S_y(s) = \frac{3}{32}h^2\delta + s\cdot\delta\cdot\frac{h}{2},$$

$$S_y(C) = S_y\left(\frac{h}{2}\right) = \frac{3}{32}h^2\delta + \frac{h}{2}\cdot\delta\frac{h}{2} = \frac{11}{32}h^2\delta$$

CE:

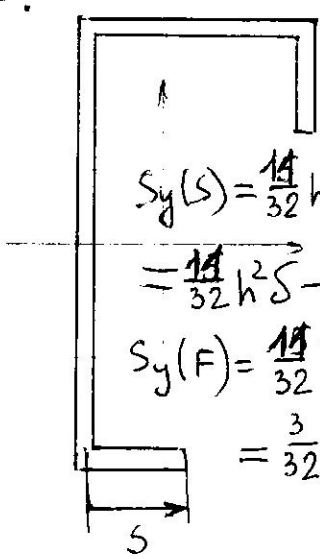


$$S_y(s) = \frac{11}{32}h^2\delta + s\cdot\delta\cdot\left(\frac{h}{2} - \frac{s}{2}\right) = \frac{11}{32}h^2\delta + s\frac{\delta}{2}(h-s)$$

$$S_y(D) = S_y\left(\frac{h}{2}\right) = \frac{11}{32}h^2\delta + \frac{h}{2}\frac{\delta}{2}\cdot\frac{h}{2} = \frac{15}{32}h^2\delta$$

$$S_y(E) = S_y(h) = \frac{11}{32}h^2\delta$$

EF:



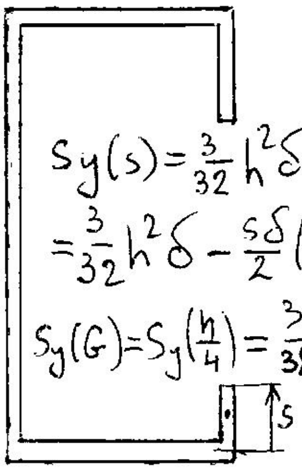
$$S_y(s) = \frac{11}{32}h^2\delta + s\cdot\delta\cdot\left(-\frac{h}{2}\right) =$$

$$= \frac{11}{32}h^2\delta - \frac{h\delta}{2}\cdot s$$

$$S_y(F) = \frac{11}{32}h^2\delta - \frac{h\delta}{2}\cdot\frac{h}{2} =$$

$$= \frac{3}{32}h^2\delta$$

FG:



$$S_y(s) = \frac{3}{32}h^2\delta + s\cdot\delta\cdot\left(-\left(\frac{h}{2} - \frac{s}{2}\right)\right) =$$

$$= \frac{3}{32}h^2\delta - \frac{s\delta}{2}(h-s)$$

$$S_y(G) = S_y\left(\frac{h}{4}\right) = \frac{3}{32}h^2\delta - \frac{h\delta}{4\cdot2}\cdot\frac{3}{4}h =$$

$$= 0$$

s. IV | Z1 c.d. $e_y = -\frac{1}{J_y}\int_0^{s_0} S_y(s)\cdot\rho\cdot ds$

Biegun w F: dla odcinka CE $\rho = \frac{h}{2}$

dla odcinka BC $\rho = h$, dla pozostałych gałęzi $\rho = 0$

$$e_y = -\frac{1}{J_y}\left(\int_0^h\left(\frac{11}{32}h^2\delta + s\frac{\delta}{2}(h-s)\right)\cdot\frac{h}{2}ds\ +\right.$$

$$\left.+\int_0^{h/2}\left(\frac{3}{32}h^2\delta + s\delta\frac{h}{2}\right)\cdot h\,ds\right) = -\frac{1}{J_y}\left(\int_0^h\left(\frac{11}{32}h^2\delta + \frac{h\delta}{2}s - \frac{\delta}{2}s^2\right)\frac{h}{2}ds\ +\right.$$

$$\left.+\frac{h^2\delta}{2}\int_0^{h/2}\left(\frac{3}{16}h + s\right)ds\right) = -\frac{1}{J_y}\left(\frac{h\delta}{4}\int_0^h\left(\frac{11}{16}h^2 + h\cdot s - s^2\right)ds\ +\right.$$

$$\left.+\frac{h^2\delta}{2}\int_0^{h/2}\left(\frac{3}{16}h+s\right)ds\right) = -\frac{1}{J_y}\left(\frac{h\delta}{4}\left[\frac{11}{16}h^2 s + \frac{h}{2}s^2 - \frac{1}{3}s^3\right]_0^h\ +\right.$$

$$\left.+\frac{h^2\delta}{2}\left[\frac{3}{16}hs + \frac{1}{2}s^2\right]_0^{\frac{h}{2}}\right) = -\frac{1}{J_y}\left(\frac{h\delta}{4}\left(\frac{11}{16}h^3 + \frac{1}{2}h^3 - \frac{1}{3}h^3\right)\ +\right.$$

$$\left.+\frac{h^2\delta}{2}\left(\frac{3}{16}\cdot\frac{h^2}{2} + \frac{1}{2}\cdot\frac{h^2}{4}\right)\right) = -\frac{1}{J_y}\left(\frac{h\delta}{4}\cdot\frac{h^3}{48}(33+24-16)\ +\right.$$

$$\left.+\frac{h^2\delta}{2}\cdot\frac{h^2}{32}(3+4)\right) = -\frac{1}{J_y}\left(\frac{41}{4\cdot 48} + \frac{7}{2\cdot 2\cdot 16}\right)h^4\delta =$$

$$= -\frac{1}{J_y}\,\frac{41+21}{4\cdot 48}h^4\delta = -\frac{31}{96}\,\frac{h^4\delta}{J_y} = -\frac{0.323\,h^4\delta}{J_y} = -\frac{0.323\,h^4\delta}{0.3535\,h^3\delta}$$

$e_y = -0.9135\,h$, $e_y + \frac{h}{2} = -0.4135\,h$